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**he relationship among FT,DTFT,**

**DFT, and z-transform**



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**Concepts**

* FT is the Fourier transform, it is mainly used in the analysis of

continuous non periodic signal, because the signal is periodic, it shall include the various frequency signal, so has the time-domain continuous aperiodic and continuous nonperiodic characteristics corresponding to the frequency-domain.

* DTFT is a discrete time Fourier transform, which is used to discrete

non-periodic sequence analysis, according to the requirements of continuous Fourier transform continuous signal in time must be the necessary and sufficient condition of integrable, so for discrete time Fourier transform, above it is used to discrete sequence must also satisfy the convergence condition of the timeline superior Numbers; Because the signal is a periodic sequence, it will contain the various frequency signal, so DTFT to discrete non periodic signal transform the spectrum of the continuous, namely a continuous cycle time discrete aperiodic corresponding frequency domain characteristics.

* Discrete Fourier Transform (Discrete Fourier Transform, abbreviated

as DFT), is the Fourier Transform in time domain and frequency domain in the form of Discrete, the signal of time domain sampling transformation of DTFT sampling frequency domain for it. Formally, at both ends (the time domain and frequency domain) sequence is limited, but in fact these two sets of sequence should be considered to be the principal value of discrete periodic signal sequence. Even the limited length of discrete signal DFT, should also be thought of as the cycle continuation of transformation. In practice usually DFT is obtained by using the fast Fourier transform.

* Z transform (Z - transformation), is a kind of mathematical

transformation of discrete sequences. Commonly used for the solution of the linear time invariant difference equation .Its position in the discrete time system, as the Laplace transform the position in continuous time systems. This approach (i.e., the Z transform of discrete time signal) analysis has become the important tool of linear time-invariant discrete time system problems. In digital signal processing, computer control system, and other fields have a wide range of applications.

**Relationship**

Arbitrary continuous cycle on the time domain signal can be decomposed into the sum of the infinite number of sine signal, in frequency domain is said to discrete the periodic signal, namely the continuous cycle corresponding to the frequency domain discrete time domain nonperiodic characteristics, which is the Fourier series expansion (FS), and it is applied to the analysis of continuous periodic signal. The FS and FT are used for continuous signal spectrum analysis tools, they are all based on Fourier series theory ask is deduced. Continuous on time domain signal in frequency domain has the characteristic of the cycle, but for periodic signal and non periodic signal in frequency domain are discrete and continuous.

Exist in nature, in addition to temperature, pressure, etc. In the continuous time signals, there are still some discrete signal, discrete signal is obtained through the continuous signal sampling, also has itself is discrete. For example, the annual rainfall in a region or an average growth rate of the signal, this kind of signal time variable for years, is not an integer DTFT is a discrete time Fourier transform, which is used to discrete non-periodic sequence analysis, according to the requirements of continuous Fourier transform continuous signal in time must be the necessary and sufficient condition of integrable, so for discrete time Fourier transform, for it above the discrete sequence of time also must satisfy the superior Numbers We know by an infinite number of the same cycle discrete signal is periodic sequence on the timeline, assuming cycle for N, namely each cycle sequence has N elements, and such a periodic sequence is infinite, infinitely many periodic sequence due to all the same, so I can only take one cycle is enough to express the whole sequence, the pumped out to represent the cycle according to the characteristic of the sequence value cycle, the said principal values sequences. Then to N corresponds to the frequency as the base frequency needed to constitute the Fourier series expansion of complex exponential sequence ek (N) = exp (j \* 2 \* PI \* k N/N), with main value sequence and complex exponential sequence correlation (by operation), each main value in each frequency spectrum component, thus show the spectrum characteristics of periodic sequence.

According to DTFT, for finite length sequences for Z transform or sequence of Fourier transform is feasible, or limited long sequences of frequency domain and complex frequency domain analysis in theory has been resolved; But for the digital system, Z transformation or sequence of Fourier transform for there exist some problems in the respects of important because the continuity of frequency variable nature (DTFT transform a continuous spectrum), is not convenient to digital computation and storage.  
 Reference DFS, similar DFS analysis method can be used to solve the above problems. Can take the finite length non-periodic sequence hypothesis for an infinitely long periodic sequence of a primary period, namely the limited long cycle continuation non-periodic sequence, after the continuation of sequence is completely DFS can be used for processing, using complex exponential sequence of fundamental frequency, the limited take long time series and draw each principal value on each frequency spectrum component to show the "principal value cycle" spectrum information.  
 Because of DFT borrowed DFS, thus assuming the sequence cycle is limitless, but when processing to range limit (main value range), to comply with limited long characteristics, which makes DFT with the cyclical. In addition, the DFT is only a week period in the representation of a finite number of discrete frequency, so it is discrete in frequency, the equivalent of DTFT transform into a continuous spectrum again after the sampling, the sampling frequency is equal to the sequence after the continuation of cycle N, or main value sequence number.

**DISCUSSION**

According to the different types of the original signal, we can put the Fourier transform is divided into four categories:

1 the periodic continuous signal Fourier Transform (Fourier Transform);

2 periodic continuous signal Fourier Series (Fourier Series);

3 non periodic signal are Discrete Time domain Fourier Transform (Discrete 'Time Fourier Transform);

4 periodic signals are Discrete Fourier Transform (Discrete 'Fourier Transform)

Z transform and discrete time Fourier transform (DTFT),Known from the type sequence of Fourier transform  
                       
Visible its value is equal to the sequence on the unit circle (= 1) z transformation, namely  
      
Or written   
Therefore calculated sequence spectrum, often to solving sequence z transformation, convergence, including the unit circle, if the z-transform will direct generation z to can.



In the continuous time signal and system, pull transform is Fourier transform is a kind of promotion, pull transformation in the time domain differential equations into algebraic equations of complex frequency domain.In the same way: in the discrete time signal and system, Z transformation can be seen as discrete Fourier transform is a kind of promotion (DFT is not always exist, can promote Z transformation, Z = r \* power, is a complex variable. When r = 1, namely the | Z | = 1, Z transform is equivalent to Fourier transform).Z transformation, the mathematical model of the discrete systems, difference equations into relatively simple algebraic equation. Therefore, the analysis of digital systems, Z transform is a powerful tool.



Some discussion about the relationship between FT, DTFT, DFT and ZT, but no clearly addressed. Lacks of self-understanding.

82